Lecture 1: 1.1 Complex Numbers

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Gerolamo Cardano (1501-1576) was the first to recognize the existence of imaginary (complex) numbers

Question: Find 2 numbers whose sum is 10 and whose product is 40

Solution: $x*(10-x)=40$

$-x^2+10x-40=0$ → solve using quadratic formula

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-10\pm\sqrt{10^2-4(-1)(-40)}}{2(-1)}$$

$$=\frac{-10\pm\sqrt{100-160}}{-2}$$

$$=\frac{-10\pm\sqrt{-60}}{-2}$$

$$=5\pm\frac{\sqrt{-60}}{\sqrt{4}}$$

$$=5\pm\sqrt{-15}$$

$\sqrt{-1}=i$

$$=5\pm\sqrt{-1}*\sqrt{15}$$

$$=\boxed{5\pm i\sqrt{15}}$$

complex number

1.1 Defining the Complex Numbers

Recall the sets of natural numbers (**N**), integers (**Z**), rational numbers (**Q**), and real numbers (**R**). Next, we can define the set of complex numbers, **C**, as:

$\mathbb{C}:=\{a+bi: a,b\,\epsilon\,\mathbb{R}\}$

real part imaginary part

OR:

"the set of all things of the form $a+bi$ where a and b are real numbers"

Notation of a complex number:

$z=a+bi\ \epsilon\ \mathbb{C}$

Re(z) = a "**real part**"

Im(z) = b "**imaginary part**"

NOTE: 2 complex numbers are only equal if and only if their real and imaginary parts agree

i is a formal symbol

- every real number $a\epsilon\mathbb{R}$ can be thought of as a complex number whose imaginary part is 0

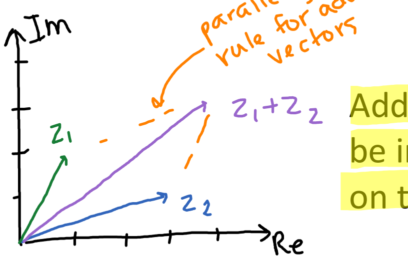
$\mathbb{R}\subseteq\mathbb{C}$ subset

Geometry:

$z_1=1+2i$

$z_2=3+i$

$z_1+z_2=4+3i$



Addition of complex numbers can be interpreted as vector addition on the complex plane

① addition

Let $z_1=a_1+b_1i$ and $z_2=a_2+b_2i$

$z_1+z_2=a_1+a_2+(b_1+b_2)i$

② Multiplication of complex #s

$z_1*z_2=a_1a_2-b_1b_2+(a_1b_2+a_2b_1)i$

eg. $i*i=(0+i)(0+i)=0-1+(0\mp0)i=-1$ $i^2=-1$ b/c $i=\sqrt{-1}$

③ Subtraction

eg. $i * i = (0+i)(0+i) = 0 - 1 + (0+0)i = -1$

③ Subtraction

$z_1 - z_2 = (a_1 + b_1 i) - (a_2 + b_2 i)$
$= a_1 - a_2 + (b_1 - b_2)i$

④ Division

$$\frac{z_1}{z_2} = \frac{a_1 + b_1 i}{a_2 + b_2 i} * \frac{a_2 - b_2 i}{a_2 - b_2 i}$$

→ we don't want a complex number in the denominator, so we multiply by the conjugate of the denominator to make the denominator a real number